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Comments on Quantum Effects in Supergravity Theories

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Abstract

We elucidate the physics underlying “anomaly mediation”, giving several alternative derivations of the formulas for gaugino and scalar masses. We stress that this phenomenon is of a type familiar in field theory, and does not represent an anomaly, nor does it depend on supersymmetry breaking and its mediation. Analogous phenomena are common in QFT and this particular phenomenon occurs also in supersymmetric theories without gravity.

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1. Introduction

The soft breaking terms in the MSSM depend on the supersymmetry breaking mechanism and on the mediation of that breaking to the fields in the MSSM. One of the most popular mediation schemes is a special case of gravity mediation which is generally referred to as “anomaly mediation” [1,2]. The anomaly mediation contribution to gaugino masses in theories with dynamical supersymmetry breaking is often the largest contribution. If the leading Kähler potential has a particular, *sequestered*, form [2], then the anomaly mediation contribution to scalar masses is the largest one and it leads to universal (if problematic) masses.

But there is much that is puzzling about these contributions:

1. The *anomaly* at issue is an anomaly in conformal transformations, which are not symmetries of the theory. What does it mean to have an anomaly in a symmetry which is not present classically? We will argue that the underlying phenomenon can be understood without relying on the conformal anomaly; in fact, we will show that the relevant phenomenon is not that of an anomaly.
2. All discussions of the problem are set within supergravity models, and most are tied to a very particular formulation of supergravity theories which uses conformal compensators, hence the relation to the conformal anomaly. We will work in components without using any particular set of auxiliary fields. We will see that *the phenomenon already arises in globally supersymmetric theories*. It has no fundamental connection to local supersymmetry, much less any particular supergravity formalism.
3. It is unclear, in the usual presentations, whether the effect should be understood in a Wilsonian effective action with a cutoff at a given energy Λ , or in a 1PI effective action. Related to this is the question whether the phenomenon is an ultraviolet (UV) or an infrared (IR) effect. We will show that the issue is a *local counterterm* which is needed in order to preserve supersymmetry. It is associated with physics *near the cutoff Λ of the Wilsonian action*, and should be thought of as an ultraviolet effect.

This note seeks to clarify the nature of these phenomena. We will explain our assertion that the effect is not an anomaly, nor is it intrinsically gravitational. We will demonstrate that it arises in theories where supersymmetry is unbroken, as well as in theories of supersymmetry breaking. The distinctive feature is the appearance of a contact term coupling the superpotential to gaugino bilinears. This term is not supersymmetric invariant. But this lack of invariance is cancelled by the lack of invariance of the measure of the light

fields. As we explain, this sort of phenomenon is already familiar in ordinary QED in two and four dimensions. We will, as a result, adopt a different terminology, referring to these phenomena as the *gaugino counterterm*.

Some experts might feel that our explanations are not new and we merely review known facts. However, a careful examination of the published literature and numerous detailed discussions with many physicists convinced us that the subject is confusing and deserves a new, clear exposition.

There are two important issues which we will not address. The first, is the question of whether the sequestered form of the Kähler potential is natural or not. The second, is whether the sequestered form leads to an acceptable phenomenology. Instead, we will focus only on the more formal field theoretic issues associated with the counterterms.

The rest of this paper is organized as follows. In the next section, we discuss at some length the *gaugino counterterm*, the crucial element of anomaly mediation. We explain that this counterterm arises from integrating out modes near the UV cutoff of the Wilsonian effective action. It can be thought of either as a contribution of the regulator fields at that scale or as a local term which is explicitly present in that action. In the latter case it seems to violate supersymmetry, but this symmetry is restored through the interactions of the light fields. We remind the reader that this phenomenon is familiar in QED both in two and four dimensions. In section three we give two derivations of the counterterms. One is a review of a standard derivation based on Pauli-Villars fields. The second considers theories in which the gauge symmetry is spontaneously broken, and the low energy effective action is completely local. In this setup, requirement of the local counterterm is almost obvious. In section four, we show that the gaugino counterterm already arises in globally supersymmetric theories. In section 5, we introduce simple models of supersymmetry breaking which incorporate features of models with dynamical supersymmetry breaking. Section six is a discussion of the scalar counterterms from this point of view. In an appendix, we discuss two and four dimensional electrodynamics in a manner which stresses the parallels to the gaugino counterterm.

2. The Gaugino Counter Term

The gaugino counterterm, the phenomenon which underlies anomaly mediation, can be understood in a variety of ways. Historically, there have been several derivations. In [3], it was observed that, at least naively, in a theory with broken supersymmetry, the presence

of heavy fields leads to one loop diagrams contributing to gaugino masses, even when the gauge coupling function is trivial, and no such term is permitted by local supersymmetry in the effective action. Regulating the diagrams with a Pauli-Villars field eliminates these terms, but this raises the question: why there are no such contributions from *massless* fields? In [1,2] it was shown that such contributions are in fact present. These derivations of gaugino masses were tied to a particular supergravity formalism, and to anomalous field redefinitions in that formalism. The authors of [2] exhibited additional contributions at two loops to scalar masses as well, beyond those expected from the local supergravity action. Reference [4] offered an explanation of this phenomenon in terms of anomalies in various field redefinitions in supergravity. They summarized their analysis in terms of non-local operators in the 1PI effective action.

These derivations are all correct, but in each case their physical significance is obscure. Our goal is to find a more satisfying conceptual setting. For this purpose, it is enough to consider supersymmetric QED; i.e. a $U(1)$ gauge theory with two chiral superfields ϕ_{\pm} with opposite $U(1)$ charges. We will include a nonzero constant W_0 in the superpotential. Because of its simplicity, we first review in this section the Pauli-Villars analysis, showing that a contact term proportional to $W^*\lambda\lambda$ is generated. Such a term arises whether or not supersymmetry is broken. *This gaugino contact term cannot arise, however, as a term in a supersymmetric effective action.* This is the would-be paradox. We explain in this section, and in the appendix, that this term is of a type quite familiar in ordinary field theory, even in QED. It does not signal the presence of an anomaly. In the next section, we briefly review the derivation using the superconformal compensator, and then provide a more transparent derivation, in a theory free of non-trivial infrared physics.

Turning to the $U(1)$ model, if supersymmetry is unbroken the cosmological constant is negative and the ground state is AdS; if supersymmetry is broken (due to some additional, hidden sector fields, say) we can use W_0 to set the cosmological constant to zero. The superpotential is

$$W = W_0 + m\phi_+\phi_- \tag{2.1}$$

and we will take, for simplicity, the constants

$$W_0 = m_{3/2}M_p^2 \quad \text{and} \quad m \tag{2.2}$$

to be real and $m \gg m_{3/2}$, such that the AdS radius $R_{AdS} = \frac{1}{m_{3/2}}$ is large compared to the inverse mass, and the space is approximately flat. The Kähler potential is simply

$$K = \bar{\phi}_+\phi_+ + \bar{\phi}_-\phi_-. \tag{2.3}$$

The supergravity potential is

$$V = e^{K/M_p^2} \left[g^{i\bar{j}} \left(\partial_i W + \frac{1}{M_p^2} \partial_i K W \right) \left(\partial_{\bar{i}} \bar{W} + \frac{1}{M_p^2} \partial_{\bar{i}} K \bar{W} \right) - 3 \frac{1}{M_p^2} W \bar{W} \right] \quad (2.4)$$

$$= -3m_{3/2}^2 M_p^2 + (m^2 - 2m_{3/2}^2)(|\phi_+|^2 + |\phi_-|^2) - m_{3/2} m (\phi_+ \phi_- + \bar{\phi}_+ \bar{\phi}_-) + \dots$$

where we neglected terms which include higher powers of the fields. The first term is the cosmological constant and the other terms lead to scalar masses and interactions. Note in particular that the interaction with nonzero W_0 leads both to contributions to scalar masses of the form $|\phi_{\pm}|^2$ and to B-terms of the form $\phi_+ \phi_- + c.c.$. The mass eigenstates are $\phi_{1,2} = \frac{1}{\sqrt{2}}(\phi_+ \mp \bar{\phi}_-)$ with eigenvalues

$$m_{1,2}^2 = m^2 - 2m_{3/2}^2 \pm m m_{3/2}. \quad (2.5)$$

A simple calculation shows that the masses of the fermionic partners of ϕ_{\pm} are not modified by the interaction with W_0 and are simply m . We see that the masses of the two bosons and the fermion are not degenerate even though supersymmetry is not broken¹.

With these masses, there is a one loop graph for the photino mass m_{λ} . Using the B-term in (2.4) we find

$$m_{\lambda} = e^2 \int \frac{d^4 p}{(2\pi)^4} \frac{m_{3/2} m^2}{(p^2 + m^2)^3} = \frac{e^2}{16\pi^2} m_{3/2} \quad (2.7)$$

and therefore there is an effective interaction

$$\frac{\alpha}{4\pi M_p^2} W_0^* \lambda \lambda \quad (2.8)$$

(λ is the photino). Note that the expression (2.7) is finite, does not need regularization and seems unambiguous.

¹ This agrees with the expression for the corresponding dimensions in the three dimensional boundary conformal field theory. The dimensions of the fermion operator and the boson operators are [5]:

$$\Delta_F = \frac{3}{2} + \frac{|m_F|}{m_{3/2}} = \frac{3}{2} + \frac{|m|}{m_{3/2}}$$

$$\Delta_{B1,2} = \frac{3}{2} + \sqrt{\frac{9}{4} + \frac{m_{1,2}^2}{m_{3/2}^2}} = \frac{3}{2} + \frac{|m|}{m_{3/2}} \pm \frac{1}{2} = \Delta_F \pm \frac{1}{2}. \quad (2.6)$$

The dimensions Δ_F and $\Delta_{B\pm} = \Delta_F \pm \frac{1}{2}$ are such that the corresponding operators are in the same supersymmetry multiplet.

This result appears paradoxical, since no such term appears in the general supergravity action [6,7]. How can loops generate terms which do not respect the symmetry of the theory, and therefore cannot be present in an invariant Lagrangian? The answer is that this term can be cancelled by a local counterterm. This is easily seen if we regulate the theory using a supersymmetric regulator like Pauli-Villars with mass Λ . The contribution of (2.8) is independent of the mass m , and similarly the contribution of the regulator is finite and is independent of Λ . Since the Pauli-Villars field contributes with the opposite sign, its contribution exactly cancels that of the fields ϕ_{\pm} (2.8), and we end up without that term!

Let us consider now the massless theory with $m = 0$. Here (2.7) vanishes, but the regulator contribution is nonzero and we are left with a term of the form of (2.8), but with the opposite sign. This is the famed “anomaly mediated” gaugino mass, in the special case of a $U(1)$ theory (and with *unbroken* supersymmetry).

To summarize, in the massive theory a term like (2.8) is not present, but it is present in the massless theory.

What is the Wilsonian action interpretation of this result? Consider first the massive theory with a UV cutoff $\Lambda \gg m$. The matter fields are much lighter than the UV cutoff and their loops are not included in the effective action. If we regularize the theory with Pauli-Villars fields with mass Λ , these fields lead to a gaugino counterterms. Therefore, this term is generated by physics near the UV cutoff Λ , but is not explicitly present in the Wilsonian action. If alternatively, we use a sharp momentum cutoff, then this term must be introduced “by hand.” The sharp momentum cutoff is not supersymmetric, and therefore it is not surprising that such a counterterm is needed². One way of thinking about this situation is that the regulated measure of the light fields is not supersymmetric and supersymmetry is restored only by adding this local counterterm.

Regardless of whether we study the system with a supersymmetric regulator, where the term arises from the physics around Λ , or with a nonsupersymmetric cutoff, where it

² The case of supersymmetric dimensional reduction was analyzed in [1] and in more detail in [8]. There it was argued that to respect the local conformal invariance of the supergravity construction with $4 - \epsilon$ spatial coordinates, certain operators with coefficients of order ϵ have to be added to the action. These operators lead to the counterterm at one loop. One can view these operators as added counterterms required for supersymmetry. Alternatively, we can leave out these operators and add the gaugino counterterm “by hand.”

is a local counterterm, we see here cancellations between contributions arising at different momentum scales.

It should be stressed that this phenomenon is not an anomaly. An *anomaly* is a lack of symmetry, which *cannot be restored with local counterterms*. Here, with some cutoffs the effective lagrangian is supersymmetric; with some it is not, but the symmetry can be restored by adding a local counterterm.

As the cutoff is further reduced to be of order m we should take into account the matter loops which cancel this contribution. Finally, for very low cutoff $\Lambda \ll m$ the theory includes only the photon multiplet and the counterterm is not present. We conclude that this gaugino counterterm is present only at energies above m , but it is absent at energies below m .

Since the counterterm is present at energies larger than m , it exists at all energies in the massless theory $m = 0$. However, we should point out that even though in this case the gaugino counterterm is present at very low energies, this does not mean that the photino is massive. The reason for that is that in massless QED the gauge coupling is renormalized to zero in the IR and the physical photino mass is proportional to the fine structure constant α which vanishes at low energies. This is consistent with the fact that massless SQED should have degenerate photons and photinos³.

It is easy to generalize this discussion in four different directions:

1. Consider SQED with several charged matter fields, flavors, with different masses. The coefficient of the counter term is proportional to the number of flavors which are lighter than the cutoff. More precisely, it is proportional to the beta function at that energy.
2. A non-Abelian gauge theory with matter fields also generates a gaugino counterterm which is proportional to its beta function at the scale of the UV cutoff. Here, unlike SQED, loops of gauge multiplets also contribute to the counterterm. Therefore, at one loop order we can identify the coefficient of the counterterm (2.8) as proportional to the one loop beta function coefficient b_0 .

³ We conclude that in AdS the photino remains massless both for nonzero and for zero m . This is consistent with general facts about AdS backgrounds. Massless gauge fields are associated with conserved currents in the boundary theory whose dimensions are fixed. The dimension of the superpartner of this current is therefore fixed by supersymmetry and cannot be renormalized. This dimension determines the mass of the gauginos and hence they must remain massless.

3. The calculation above which leads to a term proportional to the constant W_0 is easily extended to include the full W . Therefore, even if W_0 vanishes, the counterterm exists and leads to nontrivial interactions involving gauginos and scalars.
4. The previous discussion still holds when supersymmetry is spontaneously broken in flat space. In this case we also find gaugino masses of order $\alpha m_{3/2}$ with a coefficient which depends on the beta function. However, in this case the renormalization group evolution of this term is different. We can continue to integrate out modes to lower energies until we reach the gaugino mass and then this term stops running. Therefore, in that case the gauginos do receive nonzero physical mass. This is the case of interest for hidden sector anomaly mediation models.

The phenomenon that a local, gauge non-invariant term is generated by high momentum loops, is familiar in quantum field theory. For example, in the massless Schwinger model, the vacuum polarization is transverse:

$$\Pi_{\mu\nu} \sim g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}. \quad (2.9)$$

Part of the local, $g_{\mu\nu}$ term arises from high energy effects near the UV cutoff; the non-local term (and the remaining contribution proportional to $g_{\mu\nu}$) is associated with massless states in the loop. This as well as a similar phenomenon in four dimensional four photon scattering are reviewed in Appendix A.

Bagger et.al. [4] have exhibited a similar structure in supergravity theories which gives rise to the gaugino counterterm:

$$\Gamma = -\frac{g^2}{256\pi^2} \int d^2\theta 2\mathcal{E} W^\alpha W_\alpha \frac{1}{\square} (\mathcal{D}^2 - 9R) \times [4(T_R - 3T_G)R^+ - \dots]. \quad (2.10)$$

This includes the local term

$$\mathcal{L}_{ct} \sim \frac{g^2}{16\pi^2} W^* \lambda \lambda. \quad (2.11)$$

The parallel to equation (2.9) is clear.

3. A Transparent Derivation of the Counterterm

The traditional derivation of the contact term, which we review briefly below, relies on technical aspects of supergravity theory, especially the use of the superconformal compensator. Our derivation of the previous section may seem tied to a particular regularization

scheme, and further arguments are required to demonstrate that no additional local counterterms are possible. In this section, we present a very simple derivation which relies only on familiar field theory notions.

We start by reviewing the traditional derivation. The conformal compensator is a non-propagating field introduced in a supergravity formalism which begins with a conformally invariant structure, and introduces a spurion (the compensator) to maintain the symmetry. Following [1,2], we write the compensator as:

$$\Phi = 1 + F_\Phi \theta^2 \quad (3.1)$$

In terms of Φ , the relevant terms in the action are:

$$\int d^4\theta h(\bar{\phi}, e^{-V}\phi) \bar{\Phi}\Phi + \int d^2\theta (\Phi^3 W(\phi) + f(\phi) W_\alpha^2) + \text{h.c.} \quad (3.2)$$

h is related to the Kähler potential through

$$h = -3M_p^2 e^{-K/3M_p^2}. \quad (3.3)$$

For a model with broken supersymmetry and vanishing cosmological constant:

$$F_\Phi = m_{3/2}. \quad (3.4)$$

In our simple $U(1)$ theory, if we regulate with a Pauli-Villars field with mass Λ , the gauge coupling function at the scale μ is:

$$- \int d^2\theta \frac{b_0}{32\pi^2} \ln \left(\frac{\mu^2}{\Lambda^2 \Phi^{-2}} \right) W_\alpha^2. \quad (3.5)$$

where the power of Φ in the logarithm compensates for the dimension of Λ . Expanding the logarithm and doing the θ integration, generates precisely the gaugino contact term.

But there is a conceptually much simpler derivation, not tied to any particular supergravity formalism, or to anomalies in field redefinitions. Consider, again, the $U(1)$ theory, with massless chiral fields. This theory has a Higgs phase in which:

$$\phi_+ = \phi_- = v \quad (3.6)$$

(up to a phase). In this phase, the gauge symmetry is broken. The vector multiplet is massive, and there is one massless chiral multiplet, which can be described by the gauge invariant composite field $\phi_+ \phi_-$. In perturbation theory, there are no couplings of two light

fields to heavy fields, so there is no interesting infrared behavior in Feynman diagrams. The effective action for the gauge fields is necessarily local, so it has the form:

$$\int d^2\theta \left(\frac{1}{g^2} + \frac{b_0}{32\pi^2} \ln(\phi_+\phi_-/\Lambda^2) \right) W_\alpha^2. \quad (3.7)$$

(Of course, this term can be understood as reflecting the anomaly in the global symmetry under which ϕ_\pm are rotated by the same phase, or the conformal anomaly of SQED.) This corresponds to a gauge coupling function,

$$f = \frac{b_0}{32\pi^2} \ln(\phi_+\phi_-) + \text{const.} \quad (3.8)$$

Substituting this into the general supergravity action [6,7], this gauge coupling function leads to a gaugino contact term:

$$\frac{1}{M_p^2} \lambda \lambda g^{i\bar{i}} \frac{\partial f}{\partial \phi_i} (D_i W)^* = \frac{1}{16\pi^2 M_p^2} \lambda \lambda W_0^* + \dots \quad (3.9)$$

where $D_i W$ denotes the Kähler derivative of the superpotential with respect to the chiral field ϕ_i :

$$D_i W = \frac{\partial W}{\partial \phi_i} + \frac{1}{M_p^2} \frac{\partial K}{\partial \phi_i} W. \quad (3.10)$$

Unlike the discussion in the previous section, here the gaugino term is part of a supersymmetric Lagrangian.

This Higgs phase analysis makes it absolutely clear that the term is necessary for supersymmetry, and allows one to immediately write the complete action.

We see that the gaugino counterterm can be supersymmetrized either using a nonlocal action as in [4], or using a singular action like (3.8), but it cannot be supersymmetrized using a local regular action. In the previous section we worked around the origin in field space $\phi_\pm \approx 0$, and therefore we could not use expressions like (3.8). Here, in the Higgs phase, we are far from the origin, and therefore we can use this expression. More physically, the subtleties in the previous section are associated with massless particles (more precisely, particles which are much lighter than the UV cutoff). In the Higgs phase, there are no such particles and therefore these subtleties are absent and the term can be supersymmetrized by local operators.

The Higgs phase calculation indicates most strikingly that the gaugino contact term is not related to an anomaly. In a theory without charged massless fields in its Higgs phase, it is *required* by supersymmetry. We will see shortly that similar remarks apply to scalar mass terms. First, we demonstrate that the contact term arises already in globally supersymmetric theories.

4. The Counterterm in Globally Supersymmetric Theories

With the machinery of the previous section, we can see that the gaugino counterterm already arises in globally supersymmetric theories. Take the $U(1)$ model as before, but include also a Kahler potential:

$$K = \bar{\phi}_+ \phi_+ + \bar{\phi}_- \phi_- + \bar{z}z + \frac{1}{\mu^2}(\bar{\phi}_+ \phi_+ + \bar{\phi}_- \phi_-)\bar{z}z. \quad (4.1)$$

Here z is a field with a non-zero F component whose dynamics is not important here (see section 5), and μ is some energy scale assumed far lower than M_p , so gravity is irrelevant. The last term in (4.1) can arise in a more microscopic renormalizable theory from tree level exchange of massive gauge fields, or from loop effects. Again, consider the theory in its Higgs phase. The one loop effective action has the structure:

$$\mathcal{L}_0 + \frac{b_0}{16\pi^2} \int d^2\theta \ln(\phi_+ \phi_-) W_\alpha^2. \quad (4.2)$$

Solving for F_{ϕ_+} and F_{ϕ_-} gives

$$F_{\phi_+} = -\frac{1}{\mu^2} \phi_+ \bar{z} F_z \quad ; \quad F_{\phi_-} = -\frac{1}{\mu^2} \phi_- \bar{z} F_z. \quad (4.3)$$

So again, substituting in (4.2) yields a gaugino mass contact term:

$$\frac{2b_0}{16\pi^2 \mu^2} \lambda \lambda \bar{z} F_z. \quad (4.4)$$

This term has all of the features of the counterterm in supergravity theories. In the theory with $\phi_\pm = 0$, it cannot be written as part of a locally supersymmetric effective action. Its appearance is required by supersymmetry, but how it appears depends on the choice of regulator. For example, with a Pauli-Villars regulator, it may be calculated directly, but it is not generated with a momentum space cutoff or by dimensional reduction, and so must be added by hand in these cases.

An alternative derivation of the answer (4.4) which is valid around the origin $\phi_\pm \approx 0$ can be obtained by performing a field redefinition $\phi_\pm \rightarrow \phi_\pm(1 + \frac{\bar{z}z}{2\mu^2})$. This rescaling is not holomorphic but this is not a problem. The anomaly in this rescaling leads to a term proportional to $\int d^2\theta \log(1 + \frac{\bar{z}z}{\mu^2}) W_\alpha^2 \supset \frac{\bar{z}F_z}{\mu^2} \lambda \lambda$.

This derivation is easily generalized to an arbitrary Kähler potential which depends on fields in different representations of the gauge group. Then it leads to a term proportional to

$$\partial_j \left(\sum_R \frac{T_R}{d_R} \log(\det_{i\bar{i}} K_{i\bar{i}}) \right) F^j \lambda \lambda, \quad (4.5)$$

where the sum over R is over the different representations, T_R and d_R are the Casimir and dimension of R , and the indices i and \bar{i} label fields in R and \bar{R} . The term (4.5) has already been noted in [4]. However, these authors have set $M_p = 1$, and therefore did not stress that this term is independent of M_p and hence it is unrelated to gravity!

5. A setting: Supergravity Theories with Dynamical Supersymmetry Breaking

With a view to thinking about scalar contact terms, in this section we study a simple model for supersymmetry breaking. Many (but not all) models of tree level or dynamical supersymmetry breaking are described at low energies by this model or a simple variant of it. We start by considering a global supersymmetric theory and later we will couple it to supergravity and use it as a hidden sector for supersymmetry breaking.

We have a single chiral superfield z with a Kähler potential and a superpotential

$$\begin{aligned} K_{hidden} &= \mu^2 f\left(\frac{z\bar{z}}{\mu^2}\right) \\ W_{hidden} &= M^2 z + W_0. \end{aligned} \tag{5.1}$$

for some function $f(z\bar{z}/\mu^2)$. In models of dynamical supersymmetry breaking the scale μ is the dynamically generated scale of the theory, and it determines the low energy Kähler potential. In order for this effective theory to be valid, we need that the scale of supersymmetry breaking is much smaller than μ and hence we take $M \ll \mu$. Therefore, we took the Kähler potential to be independent of M . Note that in global supersymmetry the constant W_0 is not important and hence the theory has a $U(1)_R$ symmetry under which z rotates by a phase. Such a symmetry is common in models of supersymmetry breaking [9,10].

The potential derived from (5.1) is

$$V_{hidden} = \frac{M^4}{f''(z\bar{z}/\mu^2) \frac{z\bar{z}}{\mu^2} + f'(z\bar{z}/\mu^2)}. \tag{5.2}$$

If the function f is regular, the potential never vanishes and supersymmetry is broken (the behavior of f at infinity determines whether or not the theory has runaway behavior). It leads to $F_z \sim M^2$. If the minimum of the potential is at nonzero z (which is necessarily at $z \sim \mu$), then the $U(1)_R$ symmetry is spontaneously broken.⁴ Alternatively, as in the

⁴ The (3,2) model of [9] and its various relatives lead to a similar but somewhat more complicated situation. There as the analog of M is reduced, the expectation values of the low energy fields become larger rather than remaining constant as in our model.

O’Raifeartaigh model and in the model of [11] the minimum can be at $z = 0$, and then the R-symmetry is not broken. In this case it is enough to expand f and study

$$K_{hidden} = z\bar{z} - \frac{z^2\bar{z}^2}{\mu^2} \quad (5.3)$$

which leads to $F_z = M^2$ and the field z acquires a mass $m_z \sim M^2/\mu \ll M \ll \mu$.

Next we consider the gravitational corrections to these expressions focusing on the weak gravity limit $M_p \rightarrow \infty$. For the above field theoretic analysis to be meaningful we take $M, \mu \ll M_p$. Rather than taking $M_p \rightarrow \infty$ with fixed M, μ , we consider the limit with fixed gravitino mass

$$m_{3/2} \sim \frac{M^2}{M_p}, \quad (5.4)$$

and therefore we take

$$M, \mu \sim \sqrt{M_p} \rightarrow \infty. \quad (5.5)$$

The supergravity potential

$$V = e^{K/M_p^2} \left(\frac{1}{K_{z\bar{z}}} |\partial_z W + \frac{1}{M_p^2} K_z W|^2 - \frac{3}{M_p^2} |W|^2 \right) \quad (5.6)$$

can be analyzed in the limit $M_p \rightarrow \infty$ with (5.5). In order to cancel the orders M_p^2 and M_p contributions to the cosmological constant,

$$W_{hidden} = M^2 \left(z + \frac{1}{\sqrt{3}} M_p - \frac{\mu^2}{6\sqrt{3}M_p} + \mathcal{O}(1/M_p) \right) \quad (5.7)$$

(recall (5.5)). Order M_p^0 effects in the cosmological constant and in W_0 depend on the higher order corrections to K and on quantum effects in the low energy visible theory. We will not discuss them here.

Note that the constant term in W explicitly breaks the $U(1)_R$ symmetry. Since this constant is $\mathcal{O}(M_p^2)$ one should check that the previous results about the field z are not modified. Indeed, to the order we work the only difference due to the change in W and the gravitational corrections is to change the expectation value of z to

$$\langle z \rangle = z_0 = \frac{\mu^2}{2\sqrt{3}M_p} (1 + \mathcal{O}(1/M_p)) \quad (5.8)$$

but leaving the leading order result for the mass of z as in the field theory calculation. Note that in our limit the expectation value (5.8) is of order M_p^0 , but since the gravitational corrections are of order $\frac{1}{M_p^2}$ our analysis is consistent.

Finally, the gravitino mass is given by

$$m_{3/2} = \frac{1}{M_p^2} e^{\frac{K}{2M_p^2}} W(z_0) = \frac{M^2}{\sqrt{3}M_p} (1 + \mathcal{O}(1/M_p)) \quad (5.9)$$

in accord with (5.4).

6. Scalar Field Contact Terms

One of the remarkable observations of [2] is that not only are there contact terms for gauginos which seem incompatible with local supersymmetry, but there are also scalar terms. Such scalar mass terms are conceptually similar to the gaugino masses we discussed above. The lesson from our previous analysis is that despite appearance, there is really no tension between these masses and the local supersymmetry.

The most transparent way to understand it is as in our analysis of the gaugino contact term in the Higgs phase of the theory. In that case, there is no issue of non-locality and it is clear that supersymmetry demands the presence of the contact term. The same is true of the scalar masses, as we will see in this section.

The gaugino counterterm is most useful in hidden sector supergravity theories without gauge singlets. In this case the tree level couplings lead to very small gaugino masses and the gaugino counterterm is the leading contribution. Scalar masses, on the other hand, are easy to generate in all hidden sector theories using generic dimension four operators in the Kähler potential which couple the hidden and the visible sectors. In such a situation, the masses which are generated by the local counterterms are suppressed by powers of the fine structure constant, and hence they are negligible. Randall and Sundrum [2] considered a certain “sequestered” form of the Kähler potential, which might arise in some contexts (particularly in the case of separated branes). This form guarantees that the tree level terms (and also the one loop counterterms) do not lead to scalar masses. The leading contribution to the scalar masses arises from a two-loop counterterm. In the rest of this section we will describe this phenomenon using our Higgs phase language.

We divide the fields into two groups, visible sector fields ϕ (ϕ_+ and ϕ_- in our $U(1)$ example), and hidden sector fields, z . For simplicity we consider a single hidden sector field, as in section 5, and a single visible sector field ϕ . Instead of the interaction with the $U(1)$ gauge field we can have ϕ^3 interaction in the visible sector superpotential. The extension to a $U(1)$ theory is straightforward.

The sequestered Kähler potential is

$$K = -3M_p^2 \ln \left(1 - \frac{1}{3M_p^2} K_{vis}(\phi, \bar{\phi}) - \frac{1}{3M_p^2} K_{hid}(z, \bar{z}) \right). \quad (6.1)$$

For the hidden sector we use the model of section 5 and tune W_0 to have vanishing cosmological constant.

More concretely, the visible and the hidden sectors Kähler potentials K_{vis} and K_{hid} in (6.1) are such that

$$K = -3M_p^2 \ln \left(1 - \frac{1}{3M_p^2} \bar{\phi} \phi Z(\bar{\phi} \phi) - \frac{1}{3M_p^2} (\bar{z} z - \frac{1}{\mu^2} z^2 \bar{z}^2) \right) \quad (6.2)$$

and the superpotential is

$$W = W_0 + M^2 z + W_{vis}(\phi). \quad (6.3)$$

At tree level $Z(\phi, \bar{\phi}) = 1$. Radiative corrections in the visible sector change Z , but the important point is that it is independent of gravitational corrections.

At tree level $Z = 1$ and the scalar fields ϕ do not get supersymmetry breaking mass terms [2]. One can then do a calculation of the corrections to the masses, using Pauli-Villars regulators as for the gaugino masses. The Pauli-Villars fields have a non-zero supersymmetry-breaking mass (B-term). Since $\mu \ll M_p$, this is simply:

$$\Lambda m_{3/2} \phi^2 + \text{c.c.} \quad (6.4)$$

So the mass matrix for these fields is precisely that of a gauge-mediated theory (for a review, see e.g. [12]), and we can immediately read off the two loop correction to the masses of the light fields:

$$m_s^2 = -2 \left(\frac{\alpha}{16\pi^2} \right)^2 \quad (6.5)$$

Here we have given the expression when the visible sector is SQED which has two chiral superfields ϕ_{\pm} . In the case of a single ϕ with a ϕ^3 superpotential interaction α in (6.5) is replaced by the square of the cubic coupling. As for the gaugino mass, the result is independent of the Pauli-Villars mass Λ . As there, it cancels the corresponding contributions from physical heavy fields, and we are left with the Paul-Villars contribution associated with the light fields. The negative sign comes from the need to *subtract* the contribution of the Pauli-Villars fields.

All of this is precisely analogous to the behavior we saw for the gaugino mass. In the conformal compensator approach, these masses arise, as in that case, from thinking of the ultraviolet cutoff as dependent on the compensator [2]. Once more, these results can be understood in terms of the appearance in the Wilsonian effective action of a counterterm which does not respect the local supersymmetry. The lack of local SUSY invariance is needed in order to compensate the lack of invariance of the measure of the light fields.

The required supergraph calculation in this case is more challenging (one needs the terms in the lagrangian quadratic in the auxiliary fields in the gravity multiplet, for example).

A Higgs phase calculation similar to the one we used for the gaugino masses is only slightly more complicated than in that case. As we now illustrate, the scalar masses follow from straightforwardly computing the Kähler potential, and then using the supergravity action to determine the scalar potential.

In the Higgs phase, Z receives ϕ -dependent radiative corrections. Including the leading logarithms up to two loops we have

$$Z = 1 + a_1 \epsilon \ln(\bar{\phi}\phi) + a_2 \epsilon^2 \ln^2(\bar{\phi}\phi). \quad (6.6)$$

Here $\epsilon = \frac{g^2}{16\pi^2}$ or the square of the cubic coupling in the superpotential. The coefficients a_1 and a_2 can be read off of standard calculations in supersymmetry (see, e.g., [13]). In the case of SQED we have:

$$a_1 = 1 \quad ; \quad a_2 = 1 - 2b_0 \quad (6.7)$$

(that case needs several charged fields like ϕ_{\pm}) and other values for the Wess-Zumino model with the cubic superpotential.

Next, we substitute this expression for Z in the Kähler potential (6.2) and then in the supergravity scalar potential. Using the expectation value of z from (5.8) and tuning W_0 so that the cosmological constant vanishes we determine the potential for ϕ

$$\begin{aligned} V(z_0) &= e^{K/M_p^2} \left(g^{i\bar{i}} |\partial_i W + \frac{1}{M_p^2} \partial_i K W|^2 - \frac{3}{M_p^2} |W|^2 \right) \\ &= \frac{1}{\partial_\phi \partial_{\bar{\phi}} K_{vis}(\phi, \bar{\phi})} |\partial_\phi W_{vis}(\phi)|^2 \\ &\quad + m_{3/2}^2 \epsilon^2 (a_1^2 - 2a_2) |\phi|^2 \\ &\quad + m_{3/2} (\Delta_\phi \phi \partial_\phi W_{vis}(\phi) - 3W_{vis}(\phi) + c.c.) \\ &\quad + \mathcal{O}(1/M_p, \epsilon^3) \\ \Delta_\phi &= 1 - a_1 \epsilon + \mathcal{O}(\epsilon^2) \end{aligned} \quad (6.8)$$

(recall, $\mu, M \sim \sqrt{M_p}$.)

The first term in (6.8) is the potential for ϕ in the globally supersymmetric limit. The corrections represent supersymmetry breaking terms.

Consider first the scalar masses of the form $|\phi|^2$ (the second term in (6.8)). As claimed in [2], the sequestered form does not lead to tree level masses of order ϵ^0 . Also, the one loop correction of order ϵ^1 and the two loop contributions which could depend on logarithms like $\epsilon^2 \ln^2(\bar{\phi}\phi)$ and $\epsilon^2 \ln(\bar{\phi}\phi)$ vanish. We are left with a two loop mass term without logarithms. Such an answer can be extrapolated to $\phi \approx 0$ where it leads to the scalar mass square $m_s^2 = (a_1^2 - 2a_2)\epsilon^2 m_{3/2}^2$. This agrees with the expression of Randall and Sundrum for the scalar masses:

$$m_s^2 = 2b_0 \left(\frac{g^2}{16\pi^2} \right)^2 m_{3/2}^2. \quad (6.9)$$

The third term in (6.8) leads to B-terms and A-terms. The B-terms arise already at tree level (as in section 2). But the A-terms which originate from ϕ^3 terms in the superpotential arise at one loop and are of order ϵ . We expressed them in terms of the anomalous dimension Δ_ϕ .⁵ This way of writing them can be used to make contact with the formalism based on the conformal compensator.

Once again, in this formulation, the scale dependence of the mass, A-terms and B-terms is immediate. It is also clear, once more, that these terms are required by supersymmetry.

Appendix A. Two Familiar Analogs of the Gaugino Counter Term

A.1. Contact Terms in the Schwinger Model

Electrodynamics in 1+1 dimension poses many of the same issues which arise with the gaugino counterterm. A traditional way of describing mass generation in the Schwinger model is to examine the vacuum polarization diagram. The vacuum polarization itself is finite, but the diagram is superficially ultraviolet divergent, and this can lead to paradoxes. For example, it is easy to “prove” that the vacuum polarization tensor vanishes. Writing the transverse expression

$$\Pi_{\mu\nu}(q) = (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi(q^2) \quad (A.1)$$

one can take the trace:

$$\Pi_\mu^\mu = q^2 \Pi(q^2). \quad (A.2)$$

⁵ The $\mathcal{O}(\epsilon^2)$ terms in Δ_ϕ depend on $\log|\phi|^2$. These terms should be understood after performing wavefunction renormalization.

But at the level of Feynman diagrams for massless fields, this would seem to vanish since $\gamma^\mu \gamma^\nu \gamma_\mu = 0$ in two dimensions. However, the diagram is ultraviolet divergent by power counting, and introduction of a gauge-invariant regulator resolves the puzzle. For example, for dimensional regularization, $\gamma^\mu \gamma^\nu \gamma_\mu = \epsilon$ and, combined with the $1/\epsilon$ from the ultraviolet divergence, yields a finite contribution. Alternatively, with a Pauli-Villars regulator, one obtains two contributions. From the diagram with the massless fields, one obtains:

$$\Pi_{\mu\nu}^0(q) = (2q_\mu q_\nu - g_{\mu\nu} q^2) \frac{1}{\pi q^2}, \quad (\text{A.3})$$

while from the regulator diagram one obtains

$$\Pi_{\mu\nu}^\Lambda(q) = -g_{\mu\nu} q^2 \frac{1}{\pi q^2}. \quad (\text{A.4})$$

(In lightcone coordinates $\Pi_{\pm\pm}^0(q) = \frac{q_\pm}{\pi q_\mp}$, $\Pi_{\pm\mp}^0 = 0$, while $\Pi_{\pm\pm}^\Lambda(q) = 0$, $\Pi_{\pm\mp}^\Lambda = -\frac{1}{\pi}$). Note that neither result by itself is gauge invariant, but the combined expression is. Taking account of the normalization of the kinetic terms, this corresponds to a mass, e^2/π , for the physical excitation.

A few comments are in order. First, as for the gaugino counterterm, there is a local piece in this expression, arising from high energy modes, and there is a non-local term, from massless exchanges, which compensates for the lack of gauge invariance of the contact term. Second, it is important to point out that a failure of gauge invariance (breakdown of the Ward identity), can be understood, from a path integral perspective, as resulting from a lack of invariance of the measure. The naive measure, without the regulator field, violates gauge invariance. The regulated measure does not. This violation of gauge invariance *has nothing to do with whether the fields are massless or massive*. In the massive theory without the regulator, for a fermion of mass m , we would obtain a result identical to that above for Π^Λ .

Had one used a non-gauge invariant regulator, such as a momentum space cutoff, one would need to fix up the short distance part by adding a counterterm (i.e. a piece of the high energy Wilsonian action) to the contribution from the Feynman diagram.

A.2. A four dimensional example: light by light scattering

Consider now four dimensional, vector-like electrodynamics (with massless fermions). Here, there is no divergence associated with diagrams with four external photons. This is because of gauge invariance. But it is not true that the high energy behavior of these diagrams can be ignored.

Analogous to the Schwinger model, one expects that the 1PI action at low energies contains terms like

$$\mathcal{L} = \frac{(F_{\mu\nu}^2)(F_{\rho\sigma}^2)}{\square^2}. \quad (\text{A.5})$$

In momentum space, this includes couplings like

$$A_\mu^2 A_\nu^2 \quad ; \quad \frac{q_\mu A^\mu q_\nu A^\nu A_\rho^2}{q^2} \quad (\text{A.6})$$

and so on.

It is easy to see the role of short distances in the Feynman graphs. In the one loop graph, with four external gauge bosons, with polarization indices a, b, c, d , the graph behaves in the ultraviolet as:

$$\int \frac{d^4 p}{(2\pi)^4} \frac{\text{Tr}(\gamma^a \not{p} \gamma^b \not{p} \gamma^c \not{p} \gamma^d \not{p} + \gamma^a \not{p} \gamma^b \not{p} \gamma^d \not{p} \gamma^c \not{p} + \gamma^a \not{p} \gamma^c \not{p} \gamma^c \not{p} \gamma^d \not{p})}{(p^2)^4}. \quad (\text{A.7})$$

We can make the further simplification of contracting with $g_{ab}g_{cd}$. Then the integrand vanishes. However, it is necessary to introduce a regulator. In dimensional regularization, for example, the result is:

$$\frac{1}{(2\pi)^d} \int d^d p \frac{1}{p^4} 2\epsilon = \frac{2}{16\pi^2}. \quad (\text{A.8})$$

So there is a finite contact term of the type suggested above. The calculation is precisely analogous to that of the Schwinger model.

In the case of a massive field, the situation is parallel to that of the Schwinger model. Before regularization, there appears to be a local contact term. The integrand is now proportional to:

$$\begin{aligned} \int d^4 p \text{Tr} \Big[& \gamma^a (\not{p} + m) \gamma^b (\not{p} + m) \gamma^c (\not{p} + m) \gamma^d (\not{p} + m) \gamma^a (\not{p} + m) \gamma^b (\not{p} + m) \gamma^d (\not{p} + m) \gamma^c (\not{p} + m) \\ & + \gamma^a (\not{p} + m) \gamma^c (\not{p} + m) \gamma^c \not{p} \gamma^d \not{p} \Big] / (p^2 - m^2)^4. \end{aligned} \quad (\text{A.9})$$

Contracting with g_{ab} and g_{cd} as before, and working directly in four dimensions, the trace in the numerator becomes $24m^4$ (again, there is no divergence). So one is left with a finite, local interaction term:

$$\mathcal{L}_{4A} = \frac{1}{16\pi^2} A_\mu A^\mu A_\nu A^\nu. \quad (\text{A.10})$$

This term is not gauge invariant. Introducing a regulator cancels the contact term, leading to a gauge invariant result (the famous Euler-Heisenberg lagangian). In the case of a massless field, there is no such cancellation, but now there is a non-local term in the 1PI effective action, whose gauge-non-invariance cancels that of the contact term, just as in the supergravity case.

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